Big ~~Bad~~ Good Book of Mathematical Definitions

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Warning: Check Geometry Triangles

Welcome to a collection of "flashcard." What I mean by that is functions, theorems, postulates, and shortcuts to assist you in your mathematical journey of life.

1 Algebra

1.1 Basic Properties

1.1.1 Commutative Property of Addition:

Changing the order of addends does not change the sum.

$$x + y = y + x \tag{1}$$

1.1.2 Commutative Property of Multiplication:

Changing the order of factors does not change the product.

$$x \cdot y = y \cdot x \tag{2}$$

1.1.3 Associative Property of Addition:

When the addition of three or more numbers, the total/sum will be the same, even when the grouping of addends are changed.

$$x + (y + z) = (x + y) + z \tag{3}$$

1.1.4 Associative Property of Multiplication:

When performing a multiplication problem with more than two numbers, it does not matter which numbers you multiply first.

$$x \cdot (y \cdot z) = (x \cdot y) \cdot z \tag{4}$$

1.1.5 Distributive Property:

When presented with a number that is seemingly multiplied by an expression within grouping symbols (parentheses/brackets), each term within the group is multiplied by the number on the outside.

$$x \cdot (y \pm z) = (x \cdot y) \pm (x \cdot z) \tag{5}$$

1.1.6 Identity Property for Addition:

When adding 0 to any number, the result is the number itself.

$$x + 0 = x \tag{6}$$

1.1.7 Identity Property for Multiplication:

When multiplying 1 to any number, the result is the number itself.

$$x \cdot 1 = z \tag{7}$$

1.1.8 Inverse Property for Addition:

When adding a number and its inverse together, the sum will always be zero.

$$x + (-x) = 0 \tag{8}$$

1.1.9 Zero Product Property of Multiplication:

When multiplying 0 to any number, the result will always be the 0.

$$x \cdot 0 = 0 \tag{9}$$

1.2 General Formulas

1.2.1 Quadratic Formula:

$$x = \frac{-b^2 \pm \sqrt{b^2 - 4ac}}{2a} \tag{10}$$

1.3 Polynomials

1.3.1 Quadratic Polynomials:

$$\begin{aligned}
&(a+b)^2 = a^2 + 2ab + b^2 \\
&(a-b)^2 = a^2 - 2ab + b^2 \\
&a^2 - b^2 = (a+b)(a-b) \\
&a^2 + b^2 = (a+b)^2 - 2ab = (a-b)^2 + 2ab
\end{aligned} \tag{11}$$

2 Basic Geometry

2.1 Types of Angles

2.1.1 Angle

An angle is a union of two rays with a common endpoint.

2.1.2 Right Angle

A right angle is an angle with a measure of $90°$.

2.1.3 Acute Angle

An acute angle is an angle with a measure between $0°$ and $90°$.

2.1.4 Obtuse Angle

An obtuse angle is an angle with a measure between $90°$ and $180°$.

2.1.5 Complementary Angles

Two angles are complementary if the sum of their measures is $90°$.

2.1.6 Supplementary Angles

Two angles are supplementary if the sum of their measures is $180°$.

2.2 Types of Triangles

2.2.1 General Triangle

A three-sided figure.
Area: $A = \frac{1}{2}bh$
Perimeter: $P = a + b + c$
Sum of the measures of the angles is $180°$.

2.2.2 Equilateral Triangle

An equilateral triangle is a triangle that has three equal sides.

2.2.3 Isosceles Triangle

An isosceles triangle is a triangle that has two equal sides.

2.2.4 Scalene Triangle

A scalene triangle is a triangle in which all three sides are in different lengths, and all three angles are of different measures.

2.2.5 Similar Triangles

Similar Triangles are triangles that have the same shape. Their corresponding angles are equal and corresponding sides are proportional.
Example: $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$

2.2.6 Right Triangle

A triangle is a 90° angle.
Area: $A = \frac{1}{2}ab$
Perimeter: $P = a + b + c$
Pythagorean Theorem: A triangle is a right triangle if and only if $a^2 + b^2 = c^2$

2.2.7 45-90-45 Right Triangle

A right triangle who's other angles are 45°. The hypotenuse is $\sqrt{2}$ given the base and height are equal to 1.

2.2.8 30-60-90 Right Triangle

A right triangle who's other angles are 60° and 30°. The side opposite the 30° is $\frac{1}{2}$ the length of the hypotenuse.

2.3 Other 2D Shapes

2.3.1 Trapezoid

A four-sided figure with one pair of parallel sides.
Area: $A = \frac{1}{2}h(b_1 + b_2)$

2.3.2 Parallelogram

A four-sided figure with opposite sides parallel.
Area: $A = bh$

2.3.3 Rectangle

A four-sided figure with four right angles.
Area: $A = LW$
Perimeter: $P = 2L + 2W$

2.3.4 Rhombus

A four-sided figure with four equal sides.
Perimeter: $P = 4a$

2.3.5 Square

A four-sided figure with four equal sides and four right angles.
Area: $A = s^2$
Perimeter: $P = 4s$

2.3.6 Circle

Area: $A = \pi r^2$
Circumference: $C = 2\pi r$
Diameter: $d = 2r$
Value of pi: $\pi \approx 3.14$

2.3.7 Sphere

Volume: $V = \frac{4}{3}\pi r^3$
Surface Area: $s = 4\pi r^2$

2.3.8 Right Circular Cone

Volume: $V = \frac{1}{3}\pi r^2 h$
Lateral Surface Area: $S = \pi r\sqrt{r^2 + h^2}$

2.3.9 Right Circular Cylinder

Volume: $V = \pi r^2 h$
Lateral Surface Area: $S = 2\pi rh$

2.3.10 Rectangular Solid

Volume: $V = LWH$
Surface Area: $A = 2LW + 2WH + 2LH$

3 Trigonometry

In this section, you'll be presented with a flashcard based around Trigonometry. There will be the simple and the advanced, ordered by difficulty.

3.1 Domain of Basic Trigonometric Functions:

$$\begin{aligned}
&sin(x), \{x \mid x = \mathbb{R}\} \\
&cos(x), \{x \mid x = \mathbb{R}\} \\
&tan(x), \left\{x \mid x \neq \left(n + \frac{1}{2}\pi, n = \mathbb{Z}\right)\right\} \\
&csc(x), \{x \mid x \neq n\pi, n = \mathbb{Z}\} \\
&sec(x), \left\{x \mid x \neq \left(n + \frac{1}{2}\pi, n = \mathbb{Z}\right)\right\} \\
&cot(x), \{x \mid x \neq n\pi, n = \mathbb{Z}\}
\end{aligned} \tag{12}$$

3.2 Range of Basic Trigonometric Functions:

$$\begin{aligned}
&-1 \leq sin(x) \leq 1 \\
&-1 \leq cos(x) \leq 1 \\
&-\infty < tan(x) < \infty \\
&-\infty < cot(x) < \infty \\
&sec(x) \geq 1 \text{ AND } sec(x) \leq -1 \\
&csc(x) \geq 1 \text{ AND } csc(x) \leq -1
\end{aligned} \tag{13}$$

3.3 Right Triangle Definition:

$$\begin{aligned}
sin(x) &= \frac{\text{opposite}}{\text{hypotenuse}} \\
cos(x) &= \frac{\text{adjacent}}{\text{hypotenuse}} \\
tan(x) &= \frac{\text{opposite}}{\text{adjacent}} \\
csc(x) &= \frac{\text{hypotenuse}}{\text{opposite}} \\
sec(x) &= \frac{\text{hypotenuse}}{\text{adjacent}} \\
cot(x) &= \frac{\text{adjacent}}{\text{opposite}}
\end{aligned} \tag{14}$$

3.4 Degrees ↔ Radians Conversions:

$$\begin{aligned}
\text{Degrees} \cdot \frac{\pi}{180^\circ} &= \text{Radians} \\
\text{Radians} \cdot \frac{180^\circ}{\pi} &= \text{Degrees}
\end{aligned} \tag{15}$$

3.5 Tangent and Cotangent Identities:

$$\begin{aligned} tan(x) &= \frac{sin(x)}{cos(x)} \\ cot(x) &= \frac{cos(x)}{sin(x)} \end{aligned} \tag{16}$$

3.6 Reciprocal Identities:

$$\begin{aligned} csc(x) &= \frac{1}{sin(x)} \\ sec(x) &= \frac{1}{cos(x)} \\ cot(x) &= \frac{1}{tan(x)} \end{aligned} \tag{17}$$

3.7 Even - Odd Identities:

$$\begin{aligned} sin(-x) &= -sin(x) \\ cos(-x) &= cos(x) \\ tan(-x) &= -tan(x) \\ csc(-x) &= -csc(x) \\ sec(-x) &= sec(x) \\ cot(-x) &= -cot(x) \end{aligned} \tag{18}$$

3.8 Inverse Trigonometric Functions

3.8.1 Definition:

$$\begin{aligned} y = sin^{-1}(x) &\text{ is equivalent to } x = sin(y) \\ y = cos^{-1}(x) &\text{ is equivalent to } x = cos(y) \\ y = tan^{-1}(x) &\text{ is equivalent to } x = tan(y) \end{aligned} \tag{19}$$

3.8.2 Inverse Properties:

$$\begin{aligned} cos(cos^{-1}(x)) &= x \\ sin(sin^{-1}(x)) &= x \\ tan(tan^{-1}(x)) &= x \\ cos^{-1}(cos(x)) &= x \\ sin^{-1}(sin(x)) &= x \\ tan^{-1}(tan(x)) &= x \end{aligned} \tag{20}$$

3.8.3 Domain and Range of Inverse Trigonometric Functions (Respectively):

$$\begin{aligned} &y = sin^{-1}(x)\ ,\ -1 \leq x \leq 1\ ,\ -\frac{\pi}{2} \leq y \leq \frac{\pi}{2} \\ &y = cos^{-1}(x)\ ,\ -1 \leq x \leq 1\ ,\ -0 \leq y \leq \pi \\ &y = tan^{-1}(x)\ ,\ -\infty < x < \infty\ ,\ -\frac{\pi}{2} < y < \frac{\pi}{2} \end{aligned} \tag{21}$$

3.8.4 Alternate Notations:

$$\begin{aligned} &sin^{-1}(x) = \arcsin(x) \\ &cos^{-1}(x) = \arccos(x) \\ &tan^{-1}(x) = \arctan(x) \end{aligned} \tag{22}$$

3.9 Pythagorean Identities:

$$\begin{aligned} &sin^2(x) + cos^2(x) = 1 \\ &tan^2(x) + 1 = sec^2(x) \\ &1 + cot^2(x) = csc^2(x) \end{aligned} \tag{23}$$

3.10 Sum and Difference Formulas:

$$\begin{aligned} &sin(x \pm y) = sin(x)cos(y) \pm cos(x)sin(y) \\ &cos(x \pm y) = cos(x)cos(y) \mp sin(x)sin(y) \\ &tan(x \pm y) = \frac{tan(x) \pm tan(y)}{1 \mp tan(x)tan(y)} \end{aligned} \tag{24}$$

3.11 Half-Angle Formulas:

$$\begin{aligned} &sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 - cos(x)}{2}} \\ &cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 + cos(x)}{2}} \\ &tan\left(\frac{x}{2}\right) = \frac{(1 - cos(x))}{sin(x)} \end{aligned} \tag{25}$$

3.12 Double-Angle Formulas:

$$\begin{aligned} &sin(2x) = 2sin(x)cos(x) \\ &cos(2x) = cos^2(x) - sin^2(x) = 1 - 2sin^2(x) = 2cos^2(x) - 1 \\ &tan(2x) = \frac{2tan(x)}{1 - tan^2(x)} \end{aligned} \tag{26}$$

3.13 Co-Function Identities:

$$\begin{aligned}
cos\left(\frac{\pi}{2}-x\right) &= sin(x)\\
sin\left(\frac{\pi}{2}-x\right) &= cos(x)\\
tan\left(\frac{\pi}{2}-x\right) &= cot(x)\\
cot\left(\frac{\pi}{2}-x\right) &= tan(x)\\
csc\left(\frac{\pi}{2}-x\right) &= sec(x)\\
sec\left(\frac{\pi}{2}-x\right) &= csc(x)
\end{aligned} \tag{27}$$

3.14 Periodicity Identities:

$$\begin{aligned}
&sin(x \pm 2\pi) = sin(x)\\
&cos(x \pm 2\pi) = cos(x)\\
&tan(x \pm \pi) = tan(x)\\
&cot(x \pm \pi) = cot(x)\\
&sec(x \pm 2\pi) = sec(x)\\
&csc(x \pm 2\pi) = csc(x)
\end{aligned} \tag{28}$$

3.15 Sum to Product Formulas:

$$\begin{aligned}
sin(x) \pm sin(y) &= 2sin\left(\frac{x \pm y}{2}\right) cos\left(\frac{x \mp y}{2}\right)\\
cos(x) + cos(y) &= 2cos\left(\frac{x + y}{2}\right) cos\left(\frac{x + y}{2}\right)\\
cos(x) - cos(y) &= -2sin\left(\frac{x + y}{2}\right) sin\left(\frac{x - y}{2}\right)
\end{aligned} \tag{29}$$

3.16 Product to Sum Formulas:

$$\begin{aligned}
sin(x) \cdot sin(y) &= \frac{1}{2}\left[cos\,(x-y) - cos\,(x+y)\right]\\
cos(x) \cdot cos(y) &= \frac{1}{2}\left[cos\,(x-y) + cos\,(x+y)\right]\\
sin(x) \cdot cos(y) &= \frac{1}{2}\left[sin\,(x+y) + sin\,(x-y)\right]\\
cos(x) \cdot sin(y) &= \frac{1}{2}\left[sin\,(x+y) - sin\,(x-y)\right]
\end{aligned} \tag{30}$$

3.17 Law of Sines:

$$\frac{a}{sin(A)} = \frac{b}{sin(B)} = \frac{c}{sin(C)} \tag{31}$$

3.18 Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc \cdot cos(A)$$
$$A = cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right) \tag{32}$$

3.19 Law of Tangents:

$$\frac{a-b}{a+b} = \frac{tan\left(\frac{1}{2}\left(\alpha - \beta\right)\right)}{tan\left(\frac{1}{2}\left(\alpha + \beta\right)\right)}$$
$$\frac{b-c}{b+c} = \frac{tan\left(\frac{1}{2}\left(\beta - \gamma\right)\right)}{tan\left(\frac{1}{2}\left(\beta + \gamma\right)\right)} \tag{33}$$
$$\frac{a-c}{a+c} = \frac{tan\left(\frac{1}{2}\left(\alpha - \gamma\right)\right)}{tan\left(\frac{1}{2}\left(\alpha + \gamma\right)\right)}$$

3.20 Mollweide's Formula:

$$\frac{a+b}{c} = \frac{cos\left(\frac{1}{2}\left(\alpha - \beta\right)\right)}{sin\left(\frac{1}{2}\gamma\right)} \tag{34}$$